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ON THE STABILITY OF COMPRESSION SHOCK IN STREAMS OF SPONTANEOUSLY CONDENSING VAPOR

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The stability of compression shocks in streams of spontaneously condensing vapor is investigated using the concepts developed in [2]. The stability of a compression shock near the channel outlet, in which the pressure is constant, and ahead of the singular point is considered. The aim is to determine the effect of spontaneous condensation of shock stability. The boundary condition at the channel outlet was not varied, since the effect of this was fairly clearly demonstrated in [3-5]. It is shown that the presence of a singular point may result in the development of oscillatory effects.

Stability of compression shock in perfect gas flowing in channels of varying cross section was investigated in [1, 5] in one-dimensional approximation and linear formulation. The significant effect of the channel cross section, at which in a steady flow the shock is located, on shock stability was established, and the properties of the boundary condition at the outlet cross section were determined.

Gasdynamic flows with additional effects, such as blowing of gas into the channel through its side walls, flows in the presence of an electric field [2, 3] or entailing chemical reactions, etc. possess additional properties that affect the flow stability, in particular in flows with shock.

1. Let us consider in a quasi-one-dimensional approximation an adiabatic steady flow of condensing vapor in a channel of varying cross section.

The system of equations of fundamental laws of conservation and phase transformations is of the form [6]

$$A^{ij}\frac{\partial u_j}{\partial t} + B^{ij}\frac{\partial u_j}{\partial x} = F^i \quad (i, j = 1, 2, \ldots, 6) \tag{1.1}$$

$$\mathbf{B} = \begin{Vmatrix} B_1 & 0 \\ B_2 & B_3 \end{Vmatrix}, \quad \mathbf{B}_1 = \begin{Vmatrix} u_3 & 0 & \gamma u_1 \\ 0 & u_3 & u_2 \\ u_2^{-1} & 0 & u_3 \end{Vmatrix}$$

$$\mathbf{B}_2 = \begin{Vmatrix} 0 & 0 & u_4 \\ 0 & 0 & u_5 \\ 0 & 0 & u_6 \end{Vmatrix}, \quad \mathbf{B}_3 = \| u_3 \delta_{lk} \|$$

$$F^1 = -(\gamma - 1)(bu_6 E + hu_5) - \gamma u_1 u_3 (\ln A)'$$

$$F^2 = -(bu_6 + u_2 u_3 (\ln A)'), \quad F^3 = 0, \ F^4 = J - u_4 u_3 (\ln A)'$$

$$F^5 = Jr_* + au_4 - u_5 u_3 (\ln A)', \quad F^6 = Jr_* + 2au_5 - u_6 u_3 (\ln A)'$$

In these formulas u is a column vector with elements $(p, \rho, v, \omega_0, \omega_1, \omega_2)$; $\mathbf{A}$ is a unit matrix; $\mathbf{B}$ is matrix of block structure $l, k = 4, 5, 6$; p, ρ, and v are the pressure, density, and volume of vapor; ω_0 is the norm of the function of condensate particle distribution by sizes; $\omega_1 = \omega_0 \langle r \rangle$ and $\omega_2 = \omega_0 \langle r^2 \rangle$; $\langle r \rangle$ is the mean radius of condensate particles; $\langle r^2 \rangle$ is the mean square of the radius of condensate particles; A is channel cross section area, γ is the ratio of specific heats; E is the thermodynamic potential of particles undergoing phase transformation; J is the rate of condensate nuclei formation; r_* is the radius of condensate nuclei; b is a function of p and ρ determined by the specific law of phase transformations at the surface of condensate particles; h is a function of p and ρ determined by the law of interphase heat transfer; $a = b / (4\pi\rho_k)$; ρ_k is the condensed phase density ($\rho_k = \mathrm{const}$); t is the time, and x is a space coordinate. Recurrent indices imply summation and a prime indicates the total derivative with respect to x.

We assume that the steady flow considered here contains subsonic and supersonic sections. Transition from supersonic to subsonic velocity takes place continuously through the singular point or in the shock wave. Let the considered steady flow contain a shock wave. The parameters ahead of the wave and behind it are linked by relationships

$$u_i^- (\delta - u_3^-) = u_i^+ (\delta - u_3^+) \quad (i = 2, 4, 5, 6) \tag{1.2}$$

$$u_2^- u_3^- (\delta - u_3^-) - u_1^- = u_2^+ u_3^+ (\delta - u_3^+) - u_1^+$$

$$\frac{\gamma}{\gamma - 1} \frac{u_1^-}{u_2^-} + \frac{(\delta - u_3^-)^2}{2} = \frac{\gamma}{\gamma - 1} \frac{u_1^+}{u_2^+} + \frac{(\delta - u_3^+)^2}{2}$$

where superscripts minus and plus relate to parameters of the medium ahead and behind the shock wave, respectively, and δ is the shock wave velocity.

We shall investigate the flow stability in its subsonic section between the shock wave and some characteristic cross section $(x = x_b)$, for example, the channel outlet or the cross section at which transition from subsonic to supersonic velocity takes place, i.e. the singular point. Boundary conditions at that characteristic cross section may be specified in the general form as

$$\Psi\left(u_i, \frac{\partial u_i}{\partial t}, \frac{\partial u_i}{\partial x}, \ldots\right) = 0 \quad (x = x_b) \tag{1.3}$$

Let us assume that the subsonic part of the steady solution is perturbed. We linearize the system of Eqs. (1.1) and boundary conditions (1.2) and (1.3) on the assumption that unsteady additions of flow parameters (perturbations) are small and their dependence on time is defined in conformity with [7] by coefficient $\exp \lambda t$. We denote solutions of steady equations by $U = \mathrm{col}\,(\Pi, R, V, \Omega_0, \Omega_1, \Omega_2)$ and perturbations by $u = \mathrm{col}\,(p, \rho, v, \omega_0, \omega_1, \omega_2)$ (as well as the unsteady solution, since it is not subsequently used) and obtain the following system of equations:

$$\mathbf{B}^{ij} \frac{du_j}{dx} + (A^{ij}\lambda - G^{ij})\, u_j = 0 \tag{1.4}$$

Matrix $\mathbf{B}$ in Eqs. (1.4) is obtained from matrix $\mathbf{B}$ of Eqs. (1.1) by substituting for elements u the respective elements U, and matrix $\mathbf{G}$ can be respresented in the form of four blocks of 3×3 elements each

$$\mathbf{G} = \begin{Vmatrix} \mathbf{G}_1 & \mathbf{G}_2 \\ \mathbf{G}_3 & \mathbf{G}_4 \end{Vmatrix}, \quad \mathbf{G}_1 = \begin{Vmatrix} \gamma U_3^* & 0 & (U_3 \ln U_1^\gamma)' \\ 0 & U_3^* & U_2^* \\ 0 & U_3 U_3' U_2' & U_3' \end{Vmatrix}$$

$$\mathbf{G}_3 = \begin{Vmatrix} -J_p & -J_\rho & U_4^* \\ -(r_*J)_p & -(r_*J)_\rho & U_5^* \\ -(r_*^2J)_p & -(r_*^2J)_\rho & U_6^* \end{Vmatrix}, \quad \mathbf{G}_4 = \begin{Vmatrix} U_3^* & 0 & 0 \\ -a & U_3^* & 0 \\ 0 & -2a & U_3^* \end{Vmatrix}$$

where $U_n^* = U_n (\ln AU_n)'$, $n = 3,4,5,6$. Only the three elements $G^{15} = (\gamma - 1)\, h$, $G^{16} = (\gamma - 1)\, hE$, $G^{26} = b$ are nonzero in block G_2. Subscripts p and ρ denote respective partial derivatives.

At the shock the linearized equations are of the form

$$E^{ij} u_i^+ = DC^j + \xi H^j \tag{1.5}$$

where E is a matrix that also consists of four blocks

$$\mathbf{E} = \begin{Vmatrix} \mathbf{E}_1 & 0 \\ \mathbf{E}_2 & \mathbf{E}_3 \end{Vmatrix}, \quad \mathbf{E}_1 = \begin{Vmatrix} 0 & U_3^+ & U_2^+ \\ 1 & (U_3^+)^2 & 2U_2^+U_3^+ \\ \dfrac{\gamma}{(\gamma-1)\, U_2^+} & -\dfrac{\gamma U_1^+}{(\gamma-1)(U_2^+)^2} & U_3^+ \end{Vmatrix}$$

In block $\mathbf{E}_2$ the last column whose elements are U_4^+, U_5^+, U_6^+ is nonzero, and block $\mathbf{E}_3 = \| U_3^+ \delta_{lk} \|$, $l, k = 4, 5, 6$.

Vectors $\mathbf{C}$ and $\mathbf{H}$ are of the form

$$\mathbf{C} = \mathrm{col}\, ([U_2], 0, [U_3], [U_4], [U_5], [U_6])$$

$$\mathbf{H} = \mathrm{col}\, ([bU_6], U_2^+U_3^+ [U_3] (\ln A)' + U_3^+U_6^+ [b], \ [gU_6 + hU_5] [J], [aU_4 + Jr_*], [2aU_5 + Jr_*^2])$$

where brackets denote the difference between parameters behind and ahead the shock wave, i.e. $[U_j] = U_j^+ - U_j^-$; T and S are vapor temperature and entropy, respectively; , $g = bTS$; ξ is a parameter proportional to the shock wave shift, and D is a quantity defined by the equality $\delta = D \exp \lambda t$. In the derivation of formulas (1.5) equations of steady flow of the spontaneously condensing vapor, and allowance was made for the mobility of the shock.

The linearized condition at cross section $x = x_b$ is obtained in the form

$$\Psi_{u_i} u_i = 0 \quad (x = x_b) \tag{1.6}$$

The solution of system (1.4) in linearly dependent on six constants whose equations are obtained by the substitution of the derived general solution of that system into boudary conditions (1.5) and (1.6) which must be supplemented by the obvious equality

$$D = \xi\lambda \tag{1.7}$$

The question of shock wave stability can in many cases be resolved by using the algebraic equations (1.5) and (1.6) for determining the velocity of its shift in terms of λ , and then, using (1.7), determine λ.

2. Let us assume that boundary conditions (1.3) is satisfied so close to the shock that the solution cannot substantially change over such distance. For simplicity we shall consider the case in which the outlet pressure is maintained constant. Condition (1.6) then assumes the form

$$u_1^+ = 0 \quad (x = x_b) \tag{2.1}$$

and the expression for λ is of the form

$$\lambda = \sum_{i=1}^{4} \frac{Y_i}{Y}, \quad Y_1 = [V](1 + (\gamma - 1)(M^+)^2)(\ln A)' \tag{2.2}$$

$$Y_2 = -[b]\,\Omega_2^+ / R^+, \quad Y_3 = -[\Omega_2]\,(2 + (\gamma - 1)\,(M^+)^2)b^- / R^+$$

$$Y_4 = (\gamma - 1)\,[g\Omega_2 + h\Omega_1]\,(M^+)^2 / V^+$$

$$Y = ((M^+)^2 + 2 / (\gamma - 1))\,(1 - R^- / R^+) - (M^+)^2 \times (1 - V^- / V^+), \quad M^2 = V^2 R / \gamma\Pi$$

The denominator in that expression is always positive and, consequently, the sign of λ depends on the sign of the numerator whose first term is determined by the channel geometry, and the remaining depend on phase transformations and interphase heat exchange.

The first term is negative in the channel divergen part and positive in the convergent. In the absence of phase transformations and heat exchange this corresponds to the well known shock stability in the divergent part of the channel and its instability in the convergent part for a specified outlet pressure [1-5].

The second term is negative, hence it contributes to stability of the shock. The signs of remaining terms are determined by the laws of phase transformation and interphase heat exchange, and generally cannot be uniquely dermined.

As an example we had determined the steady flow of steam with spontaneous condensation, as defined by Eqs. (1.1), on the assumption that the thermal and calorific properties conform to those of a perfect gas. The following additional relationships and constants were used: the Knudsen number $b = 2\sqrt{2\pi\Pi R}\,(1 - \sqrt{T / (T + \Delta T)}$; $h = 2\pi\lambda_* \Delta T$, where $\lambda_* = 53.8 \cdot 10^{-3}$ J/m^2 deg. sec. is the thermal conductivity coefficient of steam [8]; $J = (\Pi / kT)^2 \rho_k^{-1} \times (2\sigma m / \pi)^{1/2} \exp[-2\pi r_*^2 \sigma / (3kT)]$, where $k = 1.38 \cdot 10^{-23}$ J/ deg is the Boltzmann constant, $\rho_k = 779.6$ Kg/m^3 $m \approx 3 \cdot 10^{-26}$ kg is the mass of a steam molecule, $\sigma = \sigma(\Pi)$ is the surface tension coefficient, $r_* = 2\sigma (T + \Delta T) / (\zeta \rho_k \Delta T)$, where $\zeta = \zeta(\Pi)$, is the phase transformation heat; $\gamma = 1.3$ for steam. Formulas for J and r_* were borrowed from [9] and functions $\sigma(\Pi)$ and $\zeta(\Pi)$ are approximations of data appearing in [10]. The flow at the channel inlet was assumed to be at Mach number $M(0) = 1.02$.

The behaviour of $\Lambda = \operatorname{sign} \lambda |\ln|\lambda||$ determined by formula (2.2) along the channel and the shape of the channel defined by $A(X)$ ($X = x / l$, where l is its maximum length, are shown in Fig. 1.

Pressure P (normalized with respect to pressure at the channel inlet) and the behaviour of supercooling ΔT (normalized with respect to its maximum value) in the flow of steam in a channel of specified shape $A(X)$ are shown in Fig. 1. For comparison, two versions were calculated: One without phase transformations, the other with spontaneous condensation are shown in Fig. 1 and 2 by dash and solid lines, respectively.

In the absence of phase transformations, as expected, $\lambda < 0$ in the divergent part of the channel and $\lambda > 0$ in its convergent part. However phase transformations and interphase heat exchange, which substantially affect flow parameters in the convergent part of the channel, also alter the behaviour of λ, and thus contribute to the shock stability, as can be seen from Fig. 2. It follows from curves shown in Fig. 1 by solid lines $\lambda < 0$ throughout the stream with spontaneous condensation.

Since steady slows with spontaneous condensation in channels of varying cross section were thoroughly investigated in [6], no further details of the behaviour of parameters of such flow are considered here.

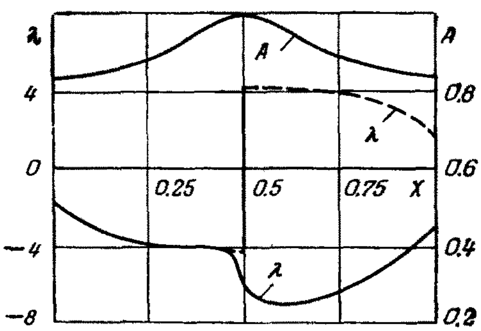

Fig. 1

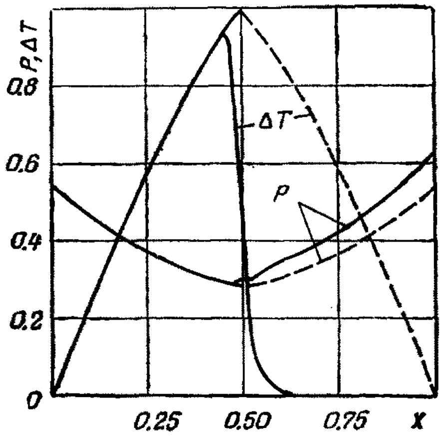

Fig. 2

3. Of interest is the case when condition (1.3) is specified at the cross section at transition through the speed of sound, induced by phase transformations [11]. If the singular point lies reasonably close to the compression shock, the solution of system (1.4) can be obtained in explicit form.

The condition at the cross section where the transition through the speed of sound takes place is obtained from the stipulation of regularity of solution of system (1.4) at the singular point.

For this we reduce the system of Eqs. (1.4) to the normal form

$$u_i' = -(\lambda A^{mj} - G^{mj})\, u_m B_{ji} / |B^{ij}| \qquad (3.1)$$

where B_{ji} is the signed minor of element B^{ij} in the expansion of determinant $|B^{ij}| = V^\nu \Pi\,(M^2 - 1)\,/\,R$. For the existence of a continuous solution passing through the singular point it is necessary that the numerator and the denominator in Eq. (3.1) simultaneously vanish. This yields

$$(\lambda A^{mj} - G^{mj})\, u_m B_{ji} = 0, \quad M = 1 \quad (x = x_b) \qquad (3.2)$$

Condition (3.2) can be considered valid when λ is small [5].
First, let us assume a linear dependence of parameters along the section Δx between the shock wave and the singular point

$$u_i(x_b) = u_i^+ + (du_i / dx)^+ \Delta x \tag{3.3}$$

In accordance with (3.1) we have

$$u_i(x_b) = u_i^+ - (\lambda A^{mj} - G^{+mj}) u_m^+ b_{ji}^+ \Delta x, \quad b_{ji}^+ = B_{ji}^+ / |B^{ij+}| \tag{3.4}$$

Then from (1.5), (3.4), and (3.2) we obtain for λ an equation of the form

$$\begin{aligned}
&\{\lambda^3 [A^{kj} A^{ms} C^n b_{sk}^+ e_{nm} \Delta x] + \lambda^2 [A^{kj} A^{ms} H^n b_{sk}^+ e_{nm} \Delta x + \\
&(G^{kj} A^{ms} + A^{kj} G^{+ms}) C^n b_{sk}^+ e_{nm} - A^{kj} C^r e_{rk} - \\
&\lambda [(G^{kj} C^r + A^{kj} H^r) e_{rk} + (G^{kj} G^{+ms} C^n + G^{kj} A^{ms} H^r + \\
&A^{ks} G^{ms} H^n) b_{sk}^+ e_{nm} \Delta x] - \\
&[G^{kj} H^r e_{rk} + G^{kj} G^{+ms} H^n b_{sk}^+ e_{nm} \Delta x]\} B_{ji} = 0 \quad (x = x_b) \\
&e_{ij} = E_{ij} / | E^{ji} |
\end{aligned}$$

where E_{ij} is the signed minor of element E^{ji} in the expansion of determinant $|E^{ij}|$.

The above cubic equation in λ may have, depending on coefficients, real, as well as complex roots. Complex roots with positive real part correspond to the unstable mode, while the negative real part shows that the perturbation damping is of a periodic kind. Pure imaginary λ indicate a possible periodic character of the flow mode [11].

REFERENCES

1. Fundamentals of Gasdynamics. Izd. Inostr. Lit., 1963.
2. Slobodkina, F. A., On the stability of compression shock in magnetogasdynamic flows in channels. PMTF, No. 1, 1970.
3. Slobodkina, F. A., Stability of quasi-onedimensional magnetohydrodynamic flows. PMM, Vol. 31, No. 3, 1967,
4. Grin', V. T., Kraiko, A. N., and Tilliaeva, N. I., On the stability of flow of perfect gas in a channel with closing compression shock and simultaneous reflection of acoustic and entropy waves from the outlet cross section. PMM, Vol. 40, No. 3, 1976.
5. Slobodkina, F. A., On the stability of subsonic gasdynamic flows. Izv. AN SSSR, MZhG, No. 1, 1978.
6. Tsiklauri, G. V., Danilin, V. S., and Seleznev, L. I., Adiabatic Two-phase Flows. Moscow, Atomizdat, 1973.
7. Brushlinskii, K. V., On the growth of solution of the mixed problem in the case of eigenfunction incompleteness. Izv. Akad. Nauk. SSSR. Ser. Matem., Vol. 23, No. 6, 1959.

8. K u t a t e l a d z e, S. S., and B o r i s h a n s k i i, V. M., (English translation), Heat Transfer Handbook. Pergamon Press, Book No. 10120, 1966.

9. F r e n k e l, Ia. I., Kinetic Theory of Fluids. Moscow-Leningrad, Izd. Akad. Nauk. SSSR, 1945.

10. V u k a l o v i c h, M. P., Thermodynamic Properties of Water and Steam. Moscow-Berlin, Mashgiz. 1958.

11. B a r s h d o r f, D. and F i l i p p o v, G. A., Analysis of certain particular operation modes of Laval nozzles with local supply of heat. Izv. Akad. Nauk SSSR, Energetika i Transport, No. 3, 1970.

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